

**Comprehensive Test Series-01**

**(Differentiation)**

**XII**

**TIME: 2hr.**

**MM: 60**

**General Instructions:**

- All Questions are compulsory.
  - Use of calculator is not permitted.
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Differentiate the functions with respect to x.

Q.1  $\cos x^3 \cdot \sin^2(x^5)$  (3)

Q.2  $2\sqrt{\cot(x^2)}$  (3)

Find  $\frac{dy}{dx}$  in the following:

Q3.  $x^3 + x^2y + xy^2 + y^3 = 81$  (3)

Q4.  $y = \sin^{-1} \left( \frac{1-x^2}{1+x^2} \right), 0 < x < 1$  (3)

Q5.  $\sqrt{e^{\sqrt{x}}}, x > 0$  (3)

Q6.  $x^{\sin x} + (\sin x)^{\cos x}$  (4)

If x and y are connected parametrically by the equations given.

Q7.  $x = a(\theta - \sin \theta), y = a(1 + \cos \theta)$  (3)

Q8.  $x = \frac{\sin^3 t}{\sqrt{\cos 2t}}, y = \frac{\cos^3 t}{\sqrt{\cos 2t}}$  (4)

Q9. If  $x = \sqrt{a^{\sin^{-1} t}}, y = \sqrt{a^{\cos^{-1} t}}$ , show that  $\frac{dy}{dx} = -\frac{y}{x}$  (3)

Find the second order derivatives of the functions

Q10. If  $e^y (x + 1) = 1$ , show that  $\frac{d^2 y}{dx^2} = \left(\frac{dy}{dx}\right)^2$  (3)

Q11. If  $y = (\tan^{-1} x)^2$ , show that  $(x^2 + 1)^2 y_2 + 2x (x^2 + 1) y_1 = 2$  (3)

Q12. Verify Rolle's theorem for the function  $y = x^2 + 2$ ,  $a = -2$  and  $b = 2$ . (3)

Q13. Verify Mean Value Theorem for the function  $f(x) = x^2$  in the interval  $[2, 4]$ . (3)

Differentiate w.r.t.  $x$  the function.

Q14.  $\cot^{-1} \left[ \frac{\sqrt{1 + \sin x} + \sqrt{1 - \sin x}}{\sqrt{1 + \sin x} - \sqrt{1 - \sin x}} \right], 0 < x < \frac{\pi}{2}$  (4)

Q15. Find  $\frac{dy}{dx}$ , if  $y = \sin^{-1} x + \sin^{-1} \sqrt{1 - x^2}$ ,  $-1 \leq x \leq 1$ . (4)

Q16. If  $\cos y = x \cos (a + y)$  with  $\cos a \neq \pm 1$ , prove that  $\frac{dy}{dx} = \frac{\cos^2(a + y)}{\sin a}$ . (4)

Q17. If  $y = e^{a \cos^{-1} x}$ ,  $-1 \leq x \leq 1$ , show that  $(1 - x^2) \frac{d^2 y}{dx^2} - x \frac{dy}{dx} - a^2 y = 0$ . Differentiate the following w.r.t.  $x$ . (4)

Q18.  $\sin^{-1} \left( \frac{2^x + 1}{1 + 4^x} \right)$  (3)