

Comprehensive Test Series-01

Limits and derivatives

TIME: 1.5hr

MM: 50

General Instructions:

- All Questions are compulsory.
 - Marks are given along with the questions individually.
 - Use of calculator is not permitted.
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Q.1 Evaluate:

$$\lim_{x \rightarrow 0} \frac{\sqrt{1+x} - 1}{x}$$

Q.2 Evaluate

$$\lim_{x \rightarrow 3} \frac{x^4 - 81}{2x^2 - 5x - 3}$$

Q.3 Evaluate:

$$\lim_{x \rightarrow 0} \frac{\sin ax}{\sin bx}, a, b \neq 0$$

Q.4 Evaluate

$$\lim_{x \rightarrow 0} \frac{\cos 2x - 1}{\cos x - 1}$$

Q.5 Evaluate

$$\lim_{x \rightarrow \frac{\pi}{2}} \frac{\tan 2x}{x - \frac{\pi}{2}}$$

Q.6 Find the derivative of the following functions from first principle

(i) $\sin x$ or $\cos x$

(ii) $x^3 - 27$

Q.7 For the function

$$(x) = \frac{x^{100}}{100} + \frac{x^{99}}{99} + \dots + \frac{x^2}{2} + x + 1.$$

Q.8 Find the derivative of

(i) $x^{-3}(5 + 3x)$

(ii) $\frac{2}{x+1} - \frac{x^2}{3x-1}$

Q.9 Find the derivative of the following functions:

$3\cot x + 5 \operatorname{cosec} x$

Q.10 $\sin(x + a)$

Q.11 $\frac{\cos x}{1 + \sin x}$

Q.12 $\frac{\sin(x + a)}{\cos x}$

Q.13 $\frac{x^2 \cos\left(\frac{\pi}{4}\right)}{\sin x}$

Q.14 $(x + \sec x)(x - \tan x)$

Q.15 Suppose $f(x) = \begin{cases} a + bx, & x < 1 \\ 4, & x = 1 \\ b - ax, & x > 1 \end{cases}$

And if $\lim_{x \rightarrow 1} f(x) = f(1)$ what are possible values of a and b?

Q.16. Compute derivative of $\cot x$