

Comprehensive Test Series-02

Sequences and Series

TIME: 2.5 hr.

MM:

General Instructions:

- All Questions are compulsory.
 - Marks are given along with the questions individually.
 - Use of calculator is not permitted.
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Q.1 In an A.P., the first term is 2 and the sum of the first five terms is one-fourth of the next five terms. Show that 20th term is -112.

Q.2 If $\frac{a^n + b^n}{a^{n-1} + b^{n-1}}$ is the A.M between a and b, then find the value of n.

Q.3 Between 1 and 31, m numbers have been inserted in such a way that the resulting sequence is an A.P. and the ratio of 7th and (m - 1)th numbers is 5:9. Find the value of m.

Q.4 The difference between any two consecutive interior angles of a polygon is 5°. If the smallest angle is 120°, find the number of the sides of the polygon.

Q.5 Evaluate $\sum_{k=1}^{11} (2 + 3^k)$

Q.6 Find the sum of the products of the corresponding terms of the sequences 2, 4, 8, 16, 32 and 128, 32, 8, 2, $\frac{1}{2}$.

Q.7 If A and G be A.M. and G.M., respectively between two positive numbers. Prove that the numbers are $A \pm \sqrt{(A+G)(A-G)}$.

Q.8 Find the sum to n terms of the series: 5 + 11 + 19 + 29 + 41 ...

Q.9 Find the sum to n terms of the series 5² + 6² + 7² + ... + 20²

Q.10 Let the sum of n, 2n, 3n terms of an A.P. be S₁, S₂ and S₃, respectively, show that S₃ = 3(S₂ - S₁)

Q.11 If $a \left(\frac{1}{b} + \frac{1}{c} \right), b \left(\frac{1}{c} + \frac{1}{a} \right), c \left(\frac{1}{a} + \frac{1}{b} \right)$ are in A.P. prove that a, b, c are in A.P.

Q.12 The ratio of the A.M. and G.M. of two positive numbers a and b, is m:n. Show that a:b = $\left(m + \sqrt{m^2 - n^2} \right) : \left(m - \sqrt{m^2 - n^2} \right)$

- Q.13 Find the sum upto n terms $.6 + .66 + .666 + \dots$
- Q.14 If S_1, S_2, S_3 are the sum of first n natural numbers, their squares and their cubes, respectively, show that $9 S_2^2 = S_3 (1 + 8S_1)$.
- Q.15 150 workers were engaged to finish a job in a certain number of days. 4 workers dropped out on second day, 4 more workers dropped out on third day and so on. It took 8 more days to finish the work. Find the number of days in which the work was completed.
- Q.16 The ratio of the sums of m and n terms of an A.P. is $m^2 : n^2$. Show that the ratio of m^{th} and n^{th} term is $(2m - 1) : (2n - 1)$
- Q.17. Find the sum to n terms
$$\frac{1}{1 \times 2} + \frac{1}{2 \times 3} + \frac{1}{3 \times 4} + \dots$$
- Q.18 The sum of the first four terms of an A.P. is 56. The sum of the last four terms is 112. If its first term is 11, then find the number of terms.
- Q.19 If a and b are the roots of $x^2 - 3x + p = 0$ and c, d are roots of $x^2 - 12x + q = 0$, where a, b, c, d form a G.P. Prove that $(q + p) : (q - p) = 17 : 15$.
- Q.20 A person writes a letter to four of his friends. He asks each one of them to copy the letter and mail to four different persons with instructions that they move the chain similarly. Assuming that the chain is not broken and that it costs 50 paise to mail one letter. Find the amount spent on the postage when 8^{th} set of letter is mailed.
- Q.21 If the first and n th term of a G.P are a and b respectively and if P is the product on n terms, prove that $p^2 = (ab)^n$
- Q.22 Insert 6 numbers between 3 and 24 such that the resulting sequence is an A.P.